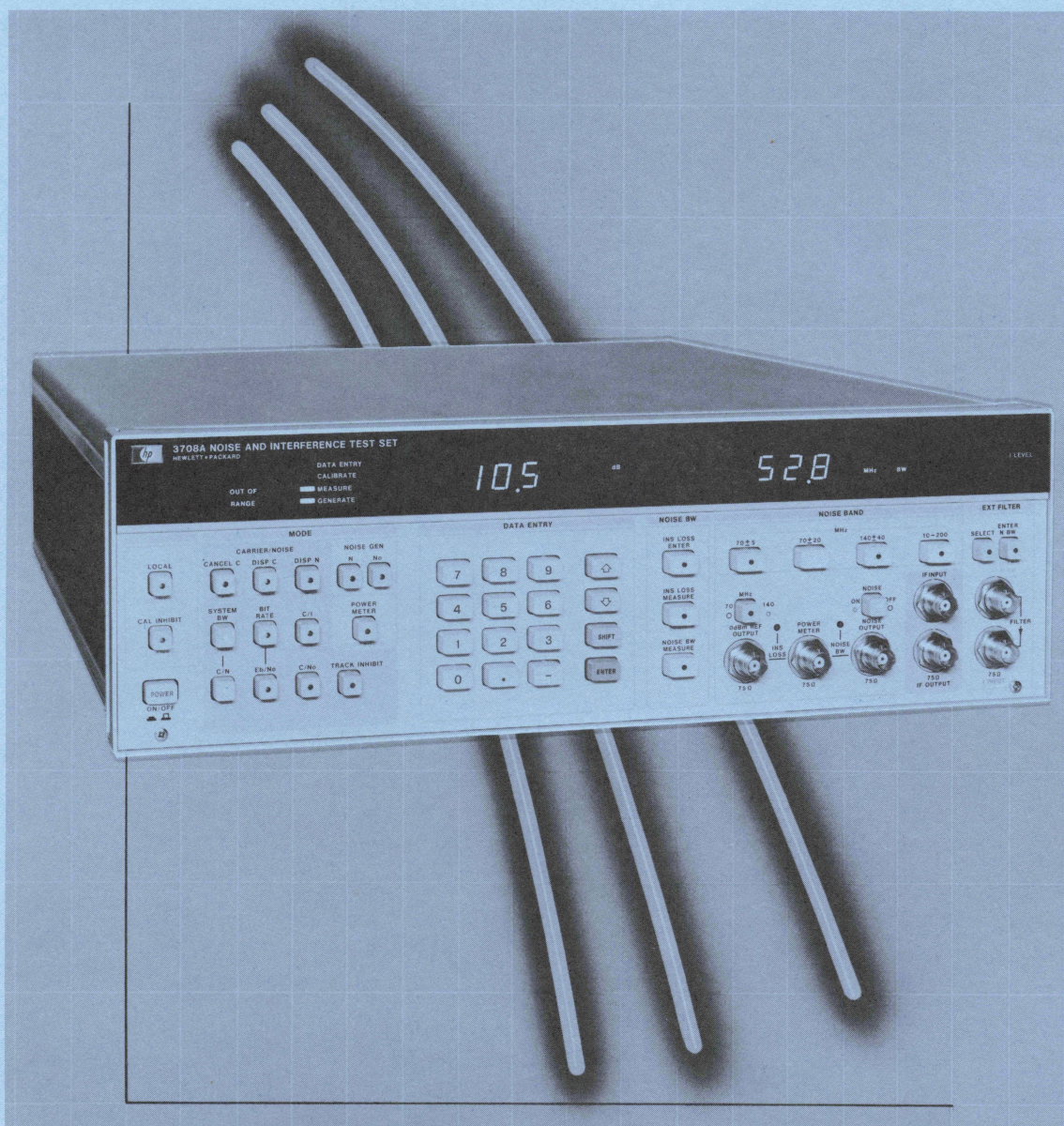

HP 3708A Noise and Interference Test Set

PRODUCT NOTE 3708-3

Determination of Residual Bit Error Ratio in Digital Microwave Systems



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**Determination of Residual Bit Error Ratio
in Digital Microwave Systems**

CONTENTS

SECTION	PAGE
1 Preface	5
2 Introduction to the "C/I Test"	6
3 Background Theory	7
3.1 Effects of Residual Noise and ISI	7
3.2 Theory of the C/I Test	8
4 Test Method - 16 QAM Modulation Scheme Example	10
4.1 C/I Ratio Measurement	10
4.2 Graphical Computation of Residual Bit Error Ratio	14
5 Accuracy of the C/I Test Method	16
6 Extension of the Technique to all Modulation Schemes	18
Appendix A: Derivation of Probability of Error for a Digital Radio System	19
Appendix B: Relationship between Symbol Error Ratio (SER) and Bit Error Ratio (BER)	24
Appendix C: Alternative Method for Providing the Interferer Tone Using an Option of the HP 3708A	27
Appendix D: Curves of Symbol Error Ratio vs C/N Ratio (for Fixed C/I Ratios) for Different Modulation Schemes	28

1. PREFACE

Residual error is a phenomenon that is present within every digital radio system, and its determination is of great importance both for initial installation, and routine maintenance of the system.

The existing methods of determining residual error ratio are both time consuming and cumbersome, requiring the digital radio to be out of service for a long period of time.

This Product Note describes a method of measuring this residual error ratio in a very short time, using the HP 3708A Noise and Interference Test Set. It is designed to reduce down-time to a minimum, whilst maintaining the accuracy associated with the old method.

2. INTRODUCTION TO THE "C/I TEST"

A pre-requisite for the successful implementation of a digital microwave radio is the accurate determination of residual Bit Error Ratio (BER), a measure of errors due to residual noise and intersymbol interference (ISI).

To measure residual BER accurately, a sufficiently large number of errors (typically more than 100) must be counted, and this can take many hours or even days at the low residual BER levels encountered in practice. For example, in a typical system, residual Bit Error Ratio would be around 10^{-10} or less.

Previously, measuring residual BER meant that a radio would have to be taken out of service for a prolonged period.

A faster and more practical method is to use the Carrier to Interference ratio (C/I) test mode of the HP 3708A Noise and Interference Test Set. In this mode, interference is added to the IF section of the digital radio receiver at selected carrier to interference ratios, using the HP 3708A, and measurements then taken of the resulting Bit Error Ratio. By using this information in a graphical extrapolation technique, an estimation of the residual BER in the radio can be derived in a fraction of the time required for the standard technique.

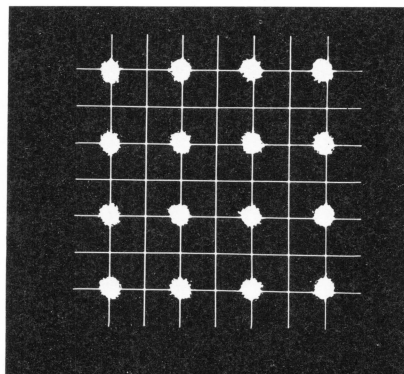
For operators of digital radios, where down-time means lost revenues, the benefits of the HP 3708A C/I test are obvious.



3. BACKGROUND THEORY

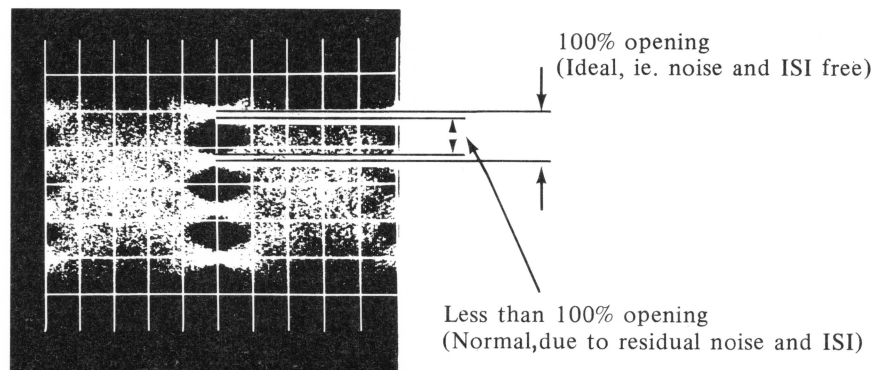
3.1 EFFECTS OF RESIDUAL NOISE AND ISI

The residual Bit Error Ratio on a digital radio system exists because of residual noise and inter-symbol interference (ISI) inherent to the system. If a sampling scope is connected to the demodulator circuit and used to view the constellation for the modulation scheme, the effects of residual noise and ISI are seen as "noise-like" clusters around the ideal constellation states. The photograph below illustrates a normal constellation for 16 QAM.



Note that in an ideal radio where there is no residual noise and no ISI, the above 16 states would be fine pin-points.

Alternatively, if the sampling scope is used to view individual symbol stream eye diagrams for a 16 QAM radio, the effect of residual noise plus ISI is seen as a reduction in the eye-opening from the ideal 100%.

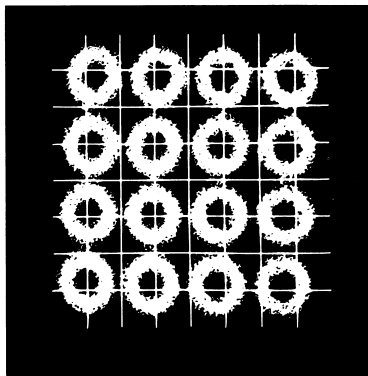


3.2 THEORY OF THE C/I TEST

If a sinusoidal interference signal is injected into the IF stage of an ideal (error-free) digital radio, at a frequency close to the radio IF, bit errors will appear at the radio's output. It can be shown that the resulting Bit Error Ratio is related to the ratio of Carrier to Interference - the C/I ratio.

If the same interference signal is injected into a practical radio, the Bit Error Ratio will be higher than that for the theoretical (ideal) radio, due to the additional effect of some residual noise and ISI.

This is shown on a constellation diagram as an annulus with the noise shifted outwards from the central points by the interferer tone:



By comparing these practical measurements with theoretically predicted results, the residual Bit Error Ratio can be determined.

The test method described in this Product Note uses this principle, in the form of a graphical extrapolation technique, to enable fast determination of residual Bit Error Ratio.

Before considering this practical technique, certain assumptions must be stated:

1. Residual noise plus ISI can be approximated to Gaussian Noise.

Refer to Appendix A for the mathematical derivation of the probability of error for a given modulation scheme, based on this assumption.

2. The coding process used in the digital radio system is Gray coding, i.e. adjacent phase states or symbols differ by only one bit.

If this last assumption is adhered to, then Bit Error Ratio is related to Symbol Error Ratio by:

$$\text{Bit Error Ratio} = 1/R \times \text{Symbol Error Ratio}$$

where R is contained in the relationship:

$$\text{Number of states in modulation scheme} = (2^R)^2$$

This expression disregards the effect of any descrambling or other "error extension" processes which will have to be considered for most digital radios.

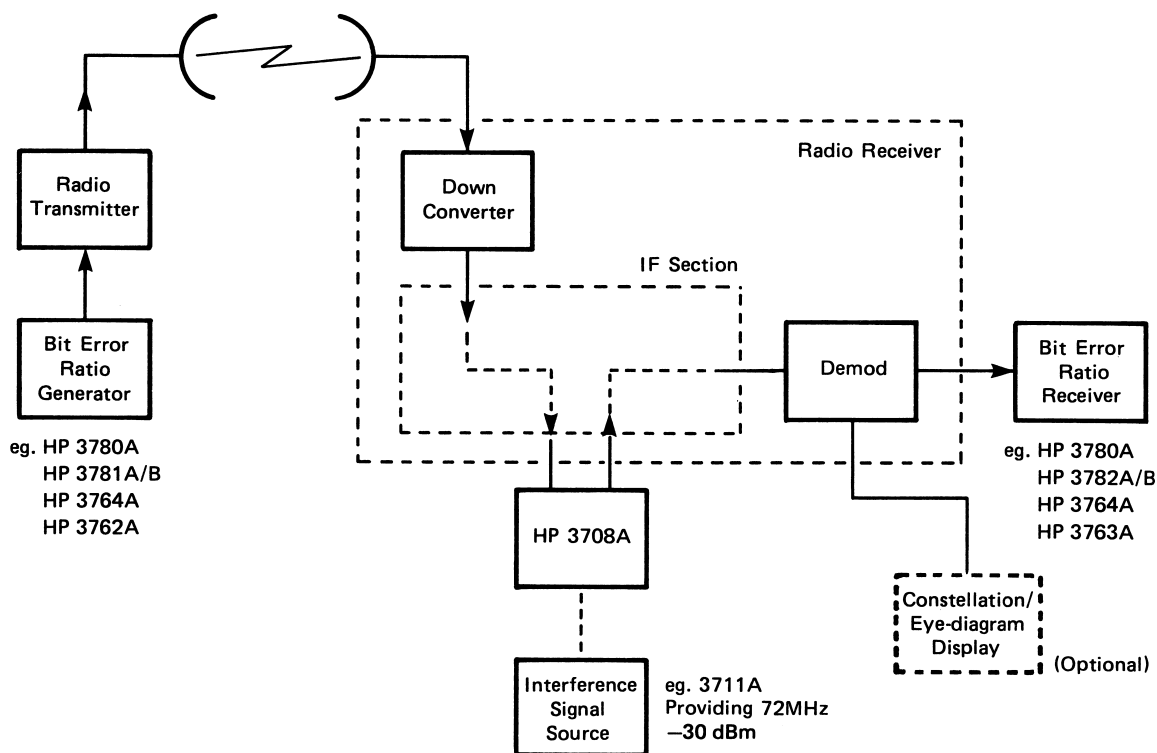
Refer to Appendix B for the derivation of the relationship between Bit Error Ratio and Symbol Error Ratio.

4. TEST METHOD - 16 QAM MODULATION SCHEME EXAMPLE

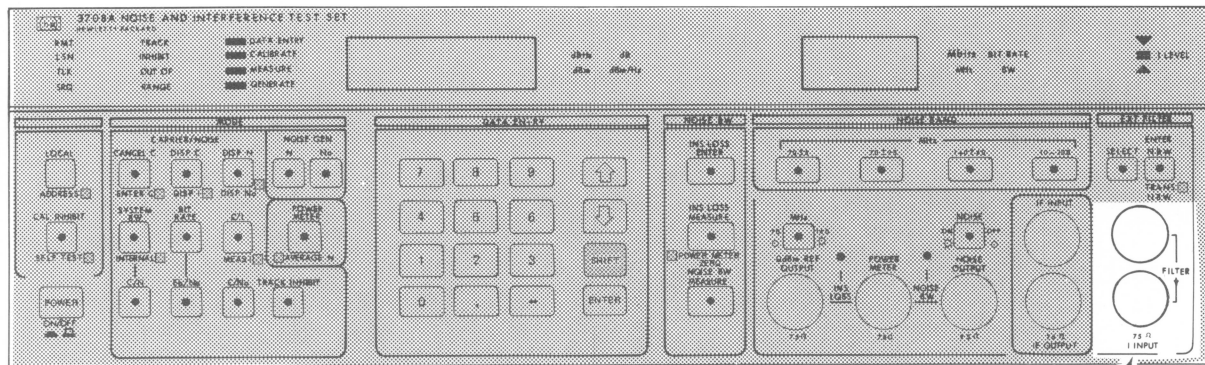
In this section the description is based upon measurements made on a digital radio using a 16 QAM modulation scheme. However, the method is suitable for other modulation schemes (see Section 6).

4.1 C/I RATIO MEASUREMENT

4.1.1 Connect the HP 3708A to any suitable access point in the IF Path of the system receiver.



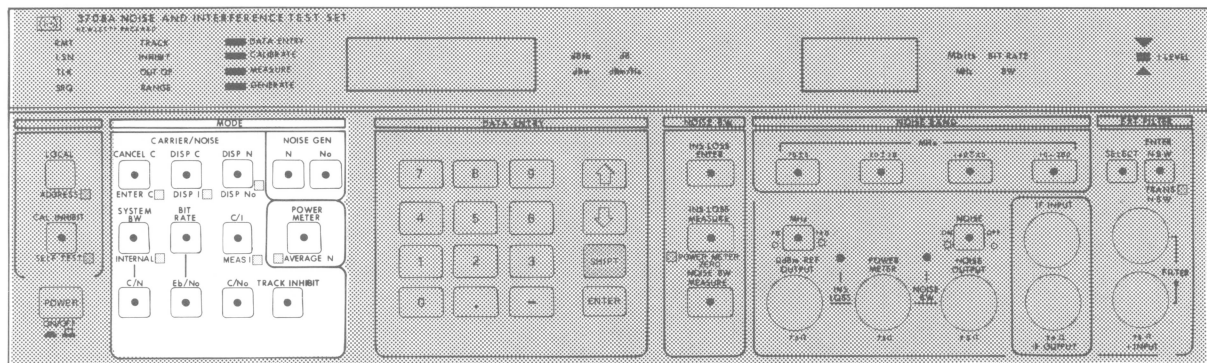
4.1.2 Supply a 72 MHz, -30 dBm sinusoidal signal to the interferer input on the front panel of the HP 3708A.



Connection of Interferer input

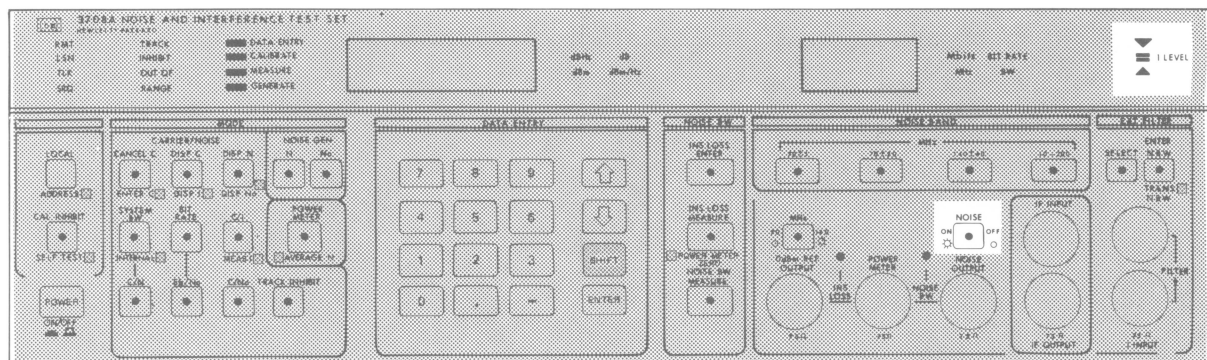
Note: A special Option is available of the HP 3708A replacing one of the 70/140 MHz reference sources by an offset frequency source, e.g. (72/140 MHz). This offset source is suitable for the C/I test. Refer to Appendix C for details.

4.1.3 Select the C/I ratio measurement on the MODE section of the Keyboard.

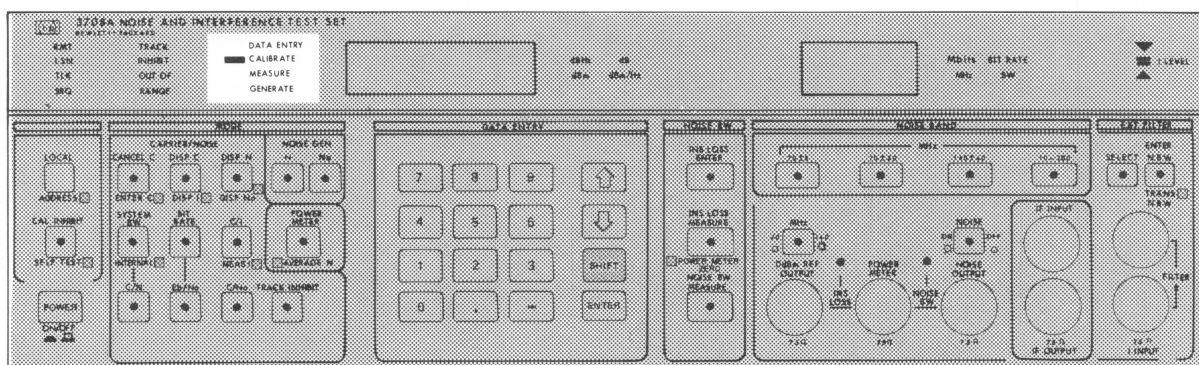


4.1.4 Adjust the level of interferer tone until the I LEVEL display shows = (If the ▼ indicator is on, reduce the inteferer level, and if the ▲ indicator is on, increase the interfeer level until the = indicator is illuminated).

4.1.5 Ensure the NOISE output is switched on (Noise key LED is illuminated). (This provides a secondary function of adding the interference tone).



4.1.6 Wait until the CALIBRATE LED extinguishes (about 15 seconds).



4.1.7. Select a suitable range of C/I ratios for the radio being tested, and quickly record the resulting BER measured on the BER receiver for each C/I ratio.

For the 16 QAM radio used for our example, a range of 17 to 26 dB was used and a table of the resulting BER values obtained (see below):

C/I Ratio (dB)	BER
17	6.4×10^{-4}
17.5	2.8×10^{-4}
18	1.2×10^{-4}
18.5	5.4×10^{-5}
19	2.5×10^{-5}
19.5	1.2×10^{-5}
20	8.0×10^{-6}
20.5	2.9×10^{-6}
21	1.6×10^{-7}
21.5	9.6×10^{-7}
22	5.8×10^{-7}
22.5	3.8×10^{-7}
23	2.3×10^{-7}
24	9.5×10^{-8}
25	5.2×10^{-8}
26	2.9×10^{-8}

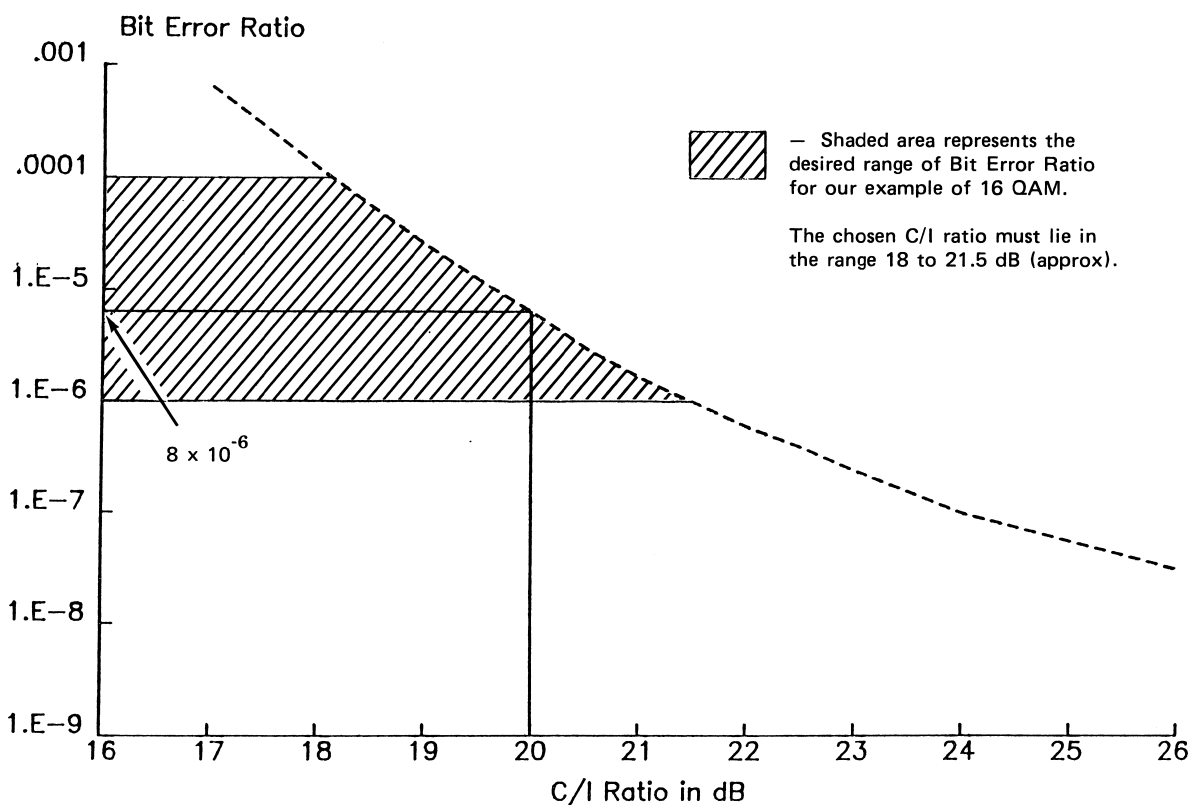
4.1.8 Select the C/I ratio which satisfies the following criteria:

- i. The chosen value of C/I ratio must coincide with one of the selected C/I ratio curves given in Appendix D for your modulation scheme.

e.g. In our 16 QAM model, the suitable values of C/I ratio are 20, 25 and 30 dB.

- ii. The resultant level of Bit Error Ratio must be high enough to be accurately measured over a relatively short period (i.e. Bit Error Ratio in the range 10^{-6} to 10^{-4}).

BER vs. C/I Ratio for a Digital Radio



In our 16 QAM example, only the 20 dB curve satisfies both criteria.

4.1.9 Measure the corresponding Bit Error Ratio accurately for the selected C/I ratio.

Note: To obtain an acceptable level of accuracy for the measurement, a minimum of 100 errors should be counted. Refer to Section 4 for accuracy of the C/I Test method.

In our example, for the chosen C/I ratio (20 dB), we accurately measured the resultant Bit Error Ratio as 8.0×10^{-6} .

4.2 GRAPHICAL COMPUTATION OF RESIDUAL BIT ERROR RATIO

Once the Bit Error Ratio has been determined with the added interferer present, the graphical technique can be used to establish the residual Bit Error Ratio.

4.2.1. Convert the measured Bit Error Ratio value to Symbol Error Ratio, using the relationship.

$$\text{Bit Error Ratio} = 1/R \times \text{Symbol Error Ratio}$$

$$\text{ie Symbol Error Ratio} = R \times \text{Bit Error Ratio}$$

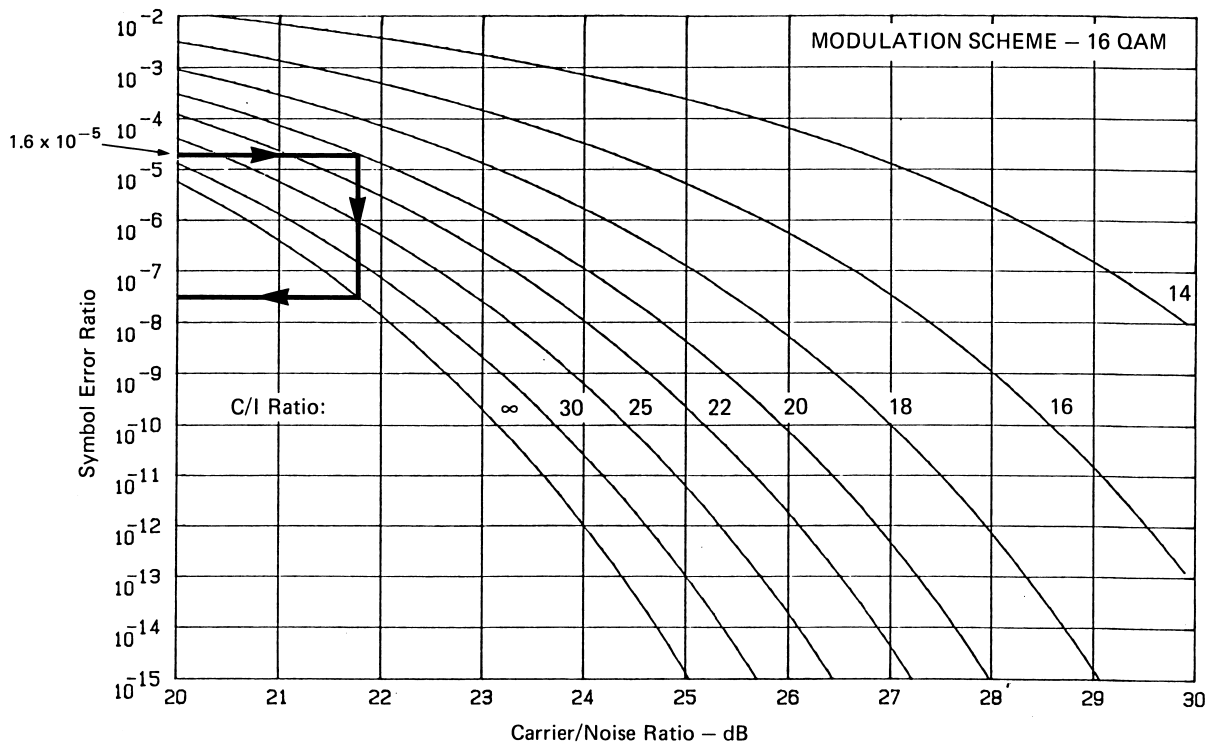
where $(2^R)^2 = M$ = the number of states in the 16 QAM modulation scheme.

$$\text{ie } R = 1/2 \log_2 M \text{ Bits/symbol}$$

Note: This relationship assumes a Gray-coded signal i.e. 1 symbol error generates only 1 bit error.

In our 16 QAM example, $R = 2$, and the equivalent Symbol Error Ratio for a C/I ratio of 20 dB is $8.0 \times 10^{-6} \times 2 = 1.6 \times 10^{-5}$.

We now use this information on the Symbol Error Ratio vs C/N ratio theoretical curves, taken from Appendix D, to relate the BER value measured on a practical radio for C/I = 20 dB, to that residual BER where I = 0 and C/I = ∞ .



4.2.2. Draw a horizontal line from the Symbol Error Ratio determined in **(4.2.1)** above.

4.2.3. At the point of intersection with the selected C/I curve (20 dB in our example) curve, draw a vertical line.

4.2.4. Where this line intersects the $C/I = \infty$ curve, extract a horizontal line back to the Symbol Error Ratio axis and read the predicted residual Symbol Error Ratio.

In our example, the residual Symbol Error Ratio is approximately 5×10^{-8} .

4.2.5. Convert this residual Symbol Error Ratio to residual Bit Error Ratio, using the relationship described in Section 3.2.

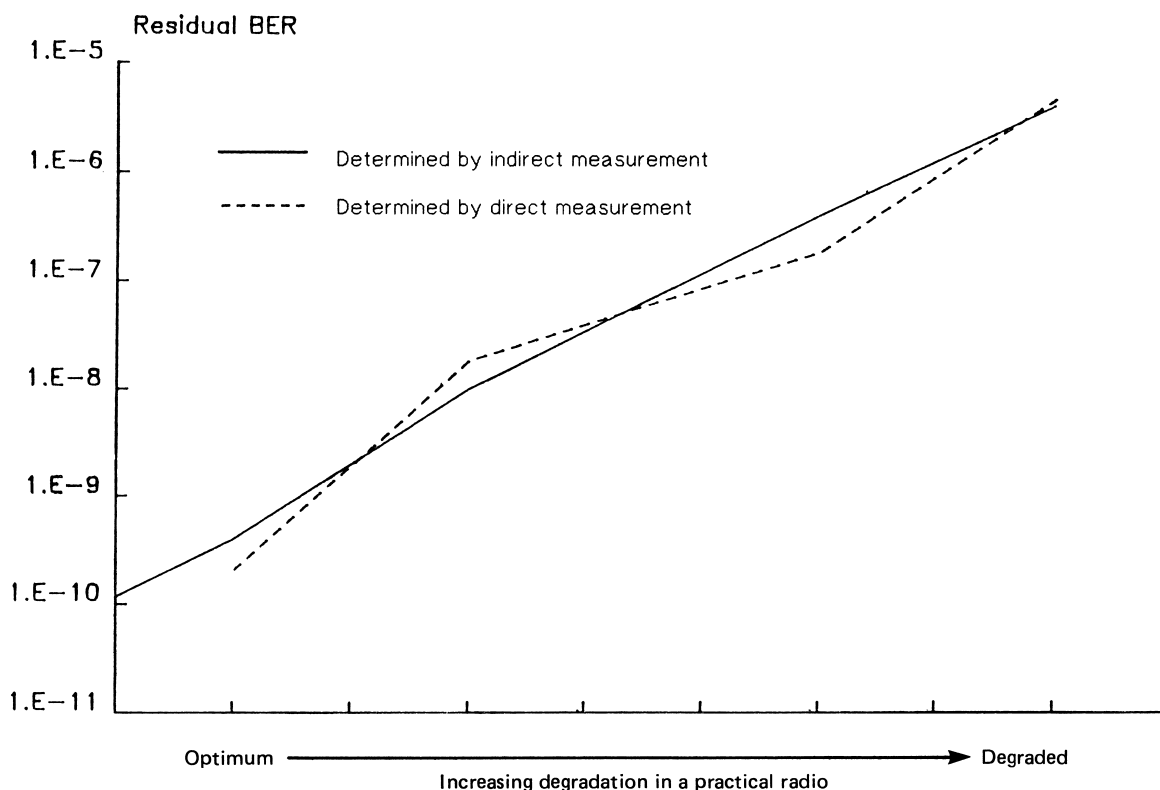
$$\text{Bit Error Ratio} = 1/R \times \text{Symbol Error Ratio}$$

In our example, the estimated residual Bit Error Ratio is:

$$5 \times 10^{-8} \times 1/2 = 2.5 \times 10^{-8}$$

5. ACCURACY OF THE "C/I TEST" METHOD

Graphical C/I method vs Direct measurement method



The above graph compares the accuracy of the C/I Test method with residual error measurements determined by the traditional manual method. The comparison is made on a digital radio with a 16 QAM modulation scheme, for a range of digital radio operating efficiencies from optimum to degraded, where the residual BER is greatly increased.

The graph shows a high degree of correlation of the results indicating the accuracy of the C/I Test method.

Accuracy of this technique is determined by 2 main factors:

1. Accuracy of the approximations used in the measurement technique:

Both residual noise and ISI are assumed to approximate to Gaussian Noise. If, however, the ISI is non-Gaussian, and is significant, then the test method described over-estimates the residual error ratio by perhaps up to an order of magnitude for practical radios.

This error could be reduced by splitting up the components contributing to residual Bit Error Ratio and treating them separately.

2. The confidence level in the accuracy of the BER measurement:

In our described test method, greater than 100 errors are counted and used in the calculation of BER by the BER receiver (Section 3.1). As we can assume that the errors approximate to a Gaussian distribution, this procedure ensures that 68% of all measured Bit Error Ratio values lie within 10% of the correct value, and 95% lie within 20% of the correct value. The use of higher error counts in determining Bit Error Ratios result in improved confidence limits.

6. EXTENSION OF THE TECHNIQUE TO ALL MODULATION SCHEMES

Our example illustration of the C/I Ratio test used curves for a 16 QAM modulation scheme. However, this method is valid for all modulation schemes, provided that the appropriate Symbol Error Ratio vs C/N ratio (for fixed C/I ratio) curves are used.

A set of curves for various common modulation schemes is shown in Appendix D.

Gray-coding is assumed in relating Bit Error Ratio to Symbol Error Ratio. If this does not apply to your digital radio, then examination of the system coding should reveal a similar expression:

$$\text{ie} \quad \text{Symbol Error Ratio} = R \times \text{Bit Error Ratio} \times X$$

where X is a multiplication factor, relating on average how many bit errors one symbol error generates. Refer to Appendix B for further explanation.

APPENDIX A: DERIVATION OF PROBABILITY OF ERROR FOR A DIGITAL RADIO SYSTEM

The following theory derives the expression for probability of error, relating to given C/N and C/I ratios. This should equip the reader with a better understanding of the test, and illustrate the principles behind the curves shown in Appendix D.

NOTE: An in-depth derivation is not provided here, and to simplify the steps assumptions are made based on other source material. Where necessary, references have been quoted.

Errors occur in digital radio transmission due to thermal noise from the components of the radio receiver, intersymbol interference (ISI) produced in the transmitter by imperfect filtering, and unwanted interference signals introduced over the microwave link.

For this analysis, the combined effects of residual thermal noise and ISI are assumed to approximate to Gaussian Noise, and the interfering signal to a sinusoidal wave.

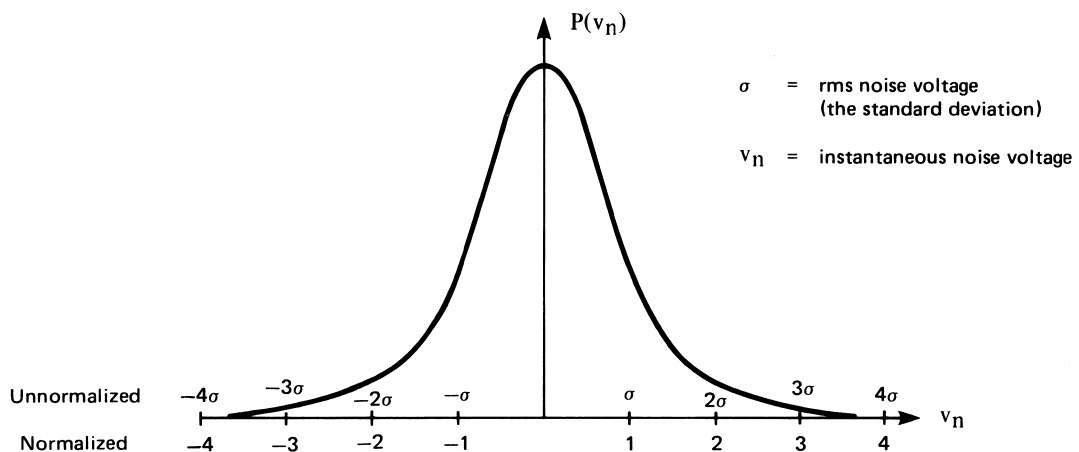
The probability density function (pdf) for Gaussian Noise, with zero dc component, is given by:

$$P(v_n) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-v_n^2 / 2\sigma^2}$$

and the normalized form, equating the rms noise voltage, σ , to 1 V is:

$$P(v_n) = \frac{1}{\sqrt{2\pi}} \times e^{-v_n^2 / 2} \quad [1.1]$$

Both can be represented as:



The area which is obtained by multiplication of the pdf by an infinitesimal width δv represents the probability that the noise has a value in width δv . The probability that the value of a noise

sample is less than a predetermined numerical value is known as the Cumulative Probability Density function (CPDF). The CPDF represents the probability that the signal $v(t)$ has a value $v_n < v_{th}$, where v_{th} has a specified value.

The normalized CPDF $F(v_{th})$ of Gaussian Noise with no dc component is given by:

$$F(v_{th}) = P(v_n < v_{th}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{v_{th}} e^{-v^2/2} dv \quad [1.2]$$

The normalized complementary CPDF, $Q(v_{th})$ can be used directly for calculation of error probability.

$$Q(v_{th}) = 1 - F(v_{th}) = \frac{1}{\sqrt{2\pi}} \int_{v_{th}}^{\infty} e^{-v^2/2} dv \quad [1.3]$$

This expression cannot be evaluated explicitly, however, but there is an equivalent polynomial approximation which can be used:

$$Q(v_{th}) = \frac{1}{\sqrt{2\pi}} \times e^{-v_{th}^2/2} \times (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) \quad [1.4]$$

$$\text{where } t = \frac{1}{1 + p v_{th}}$$

p and b_1, \dots, b_5 are constants given by:

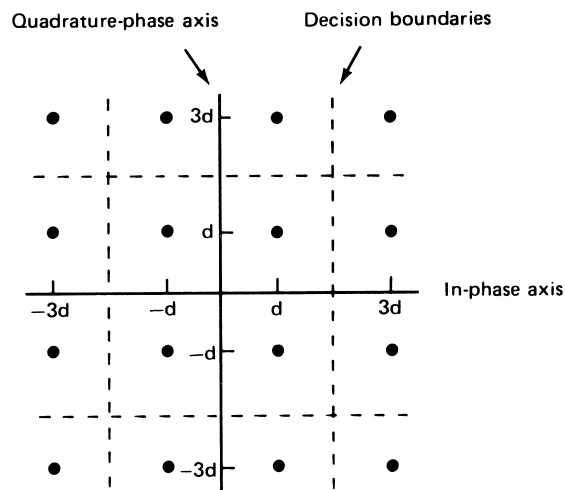
$$\begin{aligned} p &= 0.2316419 \\ b_1 &= 0.31938153 \\ b_2 &= -0.356563782 \\ b_3 &= 1.781477937 \\ b_4 &= -1.821255978 \\ b_5 &= 1.330274429 \end{aligned}$$

NOTE: The inherent accuracy of this approximation is better than 1%, down to an error probability P_e of 10^{-15} , and 10%, down to a P_e of 10^{-149} .

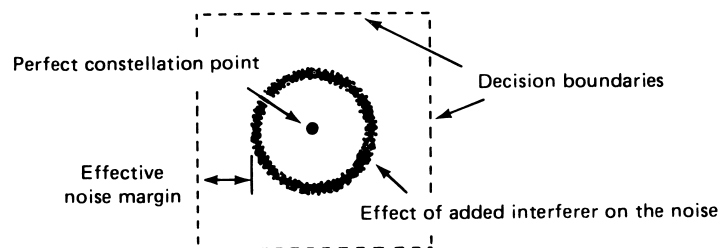
REFERENCE: Handbook of Mathematical Functions by M. Abramowitz and I.A. Stegun; published by: Dover Publications, 1965.

Deriving the Probability of Error for a Particular Modulation Scheme:

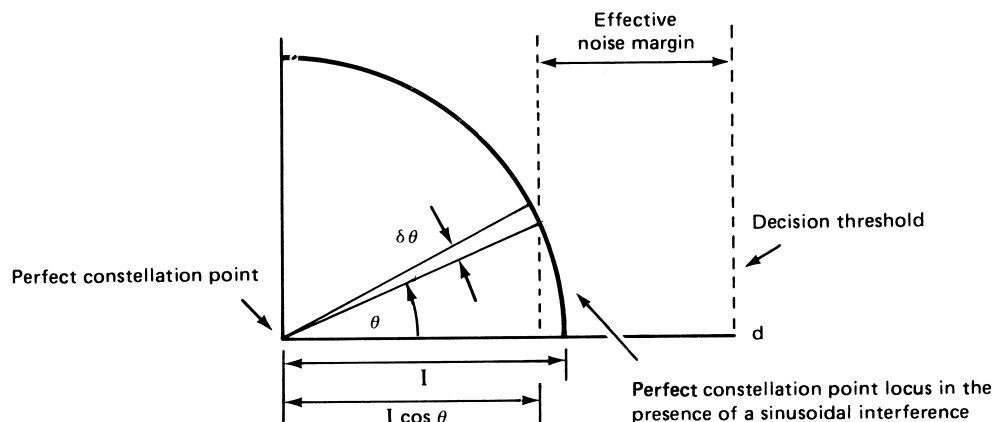
Consider the constellation diagram for a 16 QAM modulation scheme:



When a sinusoidal interferer is added to the signal, the constellation points become circles and the effective noise margin is decreased, increasing the probability that the noise voltage causes an error.



Consider the probability of the in-phase signal exceeding a single decision boundary:



The diagram shows that the effective noise margin is given by: $d - I \cos \theta$

where θ may take any value in the range 0 to 2π .

For calculation of error probability, the noise margin can be normalized (by dividing by rms noise voltage σ) and equated to v_{th} - the "threshold voltage" of equation 1.4 by:

$$v_{th} = \frac{d - I \cos \theta}{\sigma} \quad [1.5]$$

Now consider the contribution to error ratio when the angle θ is varied by a small increment $\delta \theta$.

$$\text{Incremental } Pe = Q(v_{th}) \times \frac{\delta \theta}{2\pi} \quad [1.6]$$

where Q = CPDF function (from earlier)

Considering N increments of $\delta \theta$, and summing the incremental error probability, we obtain the total error probability:

$$Pe = \sum_{n=1}^N Q(v_{th}) \times \frac{\delta \theta}{2\pi} \quad [1.7]$$

$$\text{where } v_{th} = \frac{d - I \cos \theta}{\sigma}$$

$$\theta = (n - \frac{1}{2}) \times \delta \theta$$

(The $\frac{1}{2} \delta \theta$ offset is included to give more accuracy at

the mid-point of the interval $\delta \theta = \frac{2\pi}{N}$).

This is the probability of the in-phase signal crossing a single boundary.

For our 16 QAM example, there are two in-phase decision boundaries for the inner points but only one for the outer points of the constellation. The average effect is that a factor of 3/2 must be incorporated to the total error probability.

Considering this, and substituting equation 1.4 for $Q(v_{th})$:

$$Pe = \rho \times \frac{1}{\sqrt{2\pi}} \sum_{n=1}^N \frac{1}{N} \times e^{-v_{th}^2/2} (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) \quad [1.8]$$

To evaluate the above expression, we can relate v_{th} to C/N and C/I ratios. From earlier:

$$v_{th} = \frac{d - I \cos \theta}{\sigma}$$

$$\text{where } \theta = \frac{2n-1}{2} \times \frac{2\pi}{N}$$

By expressing I in terms of C/I ratio, and σ in terms of C/N ratio, it can be shown that:

$$I = \sqrt{2} \times d \times 10^{-(C/I - \delta)/20}$$

$$\sigma = d \times 10^{-(C/N - \delta)/20} \quad [1.9]$$

where C/N, C/I are expressed in dB's, and X is a factor relating the absolute magnitude of the carrier power to different modulation schemes. For our 16 QAM example $X = 10 \log_{10} 5$.

By evaluating v_{th} in terms of C/I and C/N ratios, and substituting in equation 1.8 for v_{th} , (using the above factor for X), the probability of error for 16 QAM can then be determined.

The factors relating the average number of decision boundaries and magnitude of carrier power to equation 1.8 change for different modulation schemes. Shown in the Table below are these factors for five different modulation schemes:

Decision boundary factor		X
4PSK	2	0
8PSK	2	5.33
16PSK	2	11.19
16QAM	1.5	6.99
64QAM	1.75	13.22

Using these factors to determine the Probability of error for different modulation schemes enables the curves in Appendix D to be plotted.

APPENDIX B: RELATIONSHIP BETWEEN SYMBOL ERROR RATIO (SER) AND BIT ERROR RATIO (BER)

GRAY CODING

In a constellation diagram for a particular modulation scheme, the probability of a noise vector crossing non-adjacent decision thresholds is negligible when compared to the probability of there being a decision error into an adjacent state.

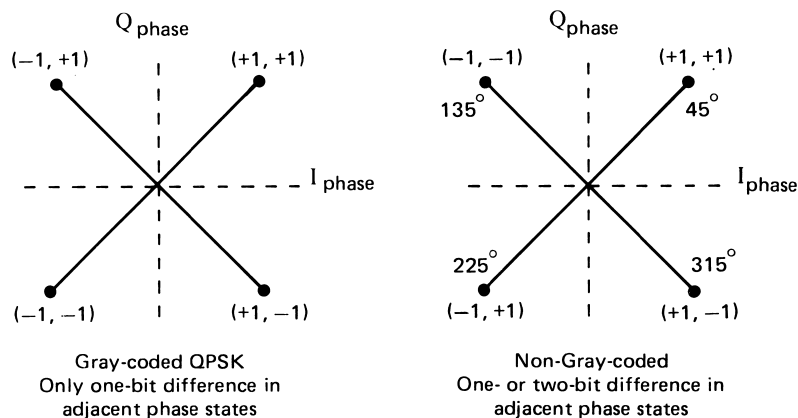
Generally:

- i. The resultant Bit Error Ratio is determined by adjacent state symbol errors.
- ii. In a Gray-coded modulation scheme, adjacent phase states or symbols differ by only one bit, whereas in non Gray coded systems, the adjacent states may differ by one or more bits.
- iii. If non Gray coding is used, then an appropriate multiplicative factor X has to be included to relate Symbol Errors to Bit Errors.
i.e. 1 Symbol Error = X Bit Errors, on average. $X \geq 1$.

QPSK Example

Consider the two QPSK coding schemes shown below:

- i. The probability of erroneous detection into 180° phase shifted states is negligible compared to the probability of decision error into adjacent (90°) phase shifted states.



- ii. In the Gray-coded system, one symbol error corresponds to one bit error, and in the non Gray-coded case, a symbol error into adjacent phase states corresponds 50% of the time, to two bit errors.

See the diagram below:

Gray Coded	Non Gray Coded
<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> $+ 45^\circ$ $+ 135^\circ$ $+ 225^\circ$ $+ 315^\circ$ </div> <div style="margin-right: 10px;"> $+1$ -1 -1 $+1$ </div> <div style="margin-right: 10px;"> $+1$ $+1$ -1 -1 </div> <div style="text-align: center;"> </div> <div> Error in adjacent phase state corresponds to one-bit error </div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> $+1$ -1 -1 $+1$ </div> <div style="margin-right: 10px;"> $+1$ -1 $+1$ -1 </div> <div style="text-align: center;"> </div> <div> Error in adjacent phase state corresponds to two-bit error </div> </div>

iii. For the non Gray-coded system shown, on average:

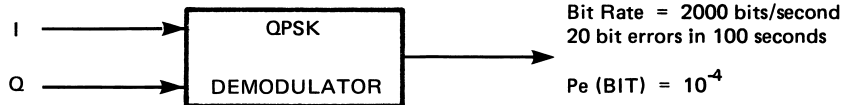
$$1 \text{ Symbol Error} = 3/2 \text{ Bit Errors}$$

Relationship between Symbol Error Ratio and Bit Error Ratio

Consider the example of a Gray-coded QPSK demodulator producing a bit rate of 2000 bit/s, from a corresponding symbol rate of 1000 symbols/s.

Symbol rate = 1000 symbols/second
10 errors in 100 seconds

$$Pe_I (\text{SYMBOL}) = 10^{-4}$$



Symbol rate = 1000 symbols/second
10 errors in 100 seconds

$$Pe_Q (\text{SYMBOL}) = 10^{-4}$$

QPSK MODULATION SCHEME

At the input to the demodulator, the Symbol Error Ratio for the in-phase (I) or quadrature-phase (Q) streams is 10^{-4} , and at the output of the demodulator, there are on average, 20 erroneous bits in a 100 second interval, (Gray-coding is assumed, i.e. multiplication factor X is 1).

The Bit Error Ratio (BER) of the demodulator output is calculated using the relationship:

$$BER = \frac{\text{number of bits in error}}{\text{total number of bits}} = \frac{20}{2000 \text{ bit/s} \times 100 \text{ s}} = 10^{-4}$$

Thus for Gray-coded QPSK;

$$(\text{Symbol Error Ratio})_I = (\text{Symbol Error Ratio})_Q = BER.$$

Similar derivations may be performed for any QAM modulation scheme.

General expression for M-ary QAM is:

$$\text{Bit Error Ratio} = (1/R) \times \text{Symbol Error Ratio} \times X$$

where $(2^R)^2 = M$ = the number of states in the modulation scheme.

X = "Non Gray-coding" multiplicative factor.

So for example, Gray-coded 16 QAM gives:

$$\text{Bit Error Ratio} = 1/2 \times \text{Symbol Error Ratio}$$

and Gray-coded 64 QAM gives:

$$\text{Bit Error Ratio} = 1/3 \times \text{Symbol Error Ratio}$$

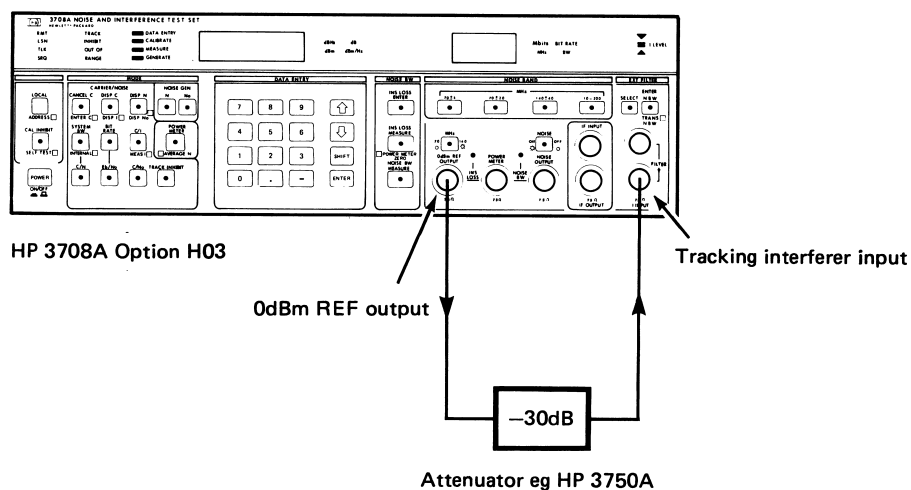
There may be an additional requirement to incorporate a factor into the above relationship to allow for the effects of error extension processes in the digital radio receiver. For example, such a factor may be determined by analysing the error extension processes related to the descrambler.

APPENDIX C: ALTERNATIVE METHOD FOR PROVIDING THE INTERFERER TONE USING AN OPTION OF THE HP 3708A

As an alternative to using an external signal source supplying a 72 MHz sinusoidal wave as an interfering tone, the internal reference output of the HP 3708A can be utilized.

If the 72 MHz Option H03 of the HP 3708A is purchased, as an alternative to the standard 70 MHz HP 3708A, the 0 dBm 72 MHz reference output can be connected, via an attenuator (e.g. HP 3750A), to the interferer input. By selecting a 30 dB attenuation, a -30 dBm, 72 MHz signal will be supplied to the interferer input. This provides a direct substitute for an external signal generator.

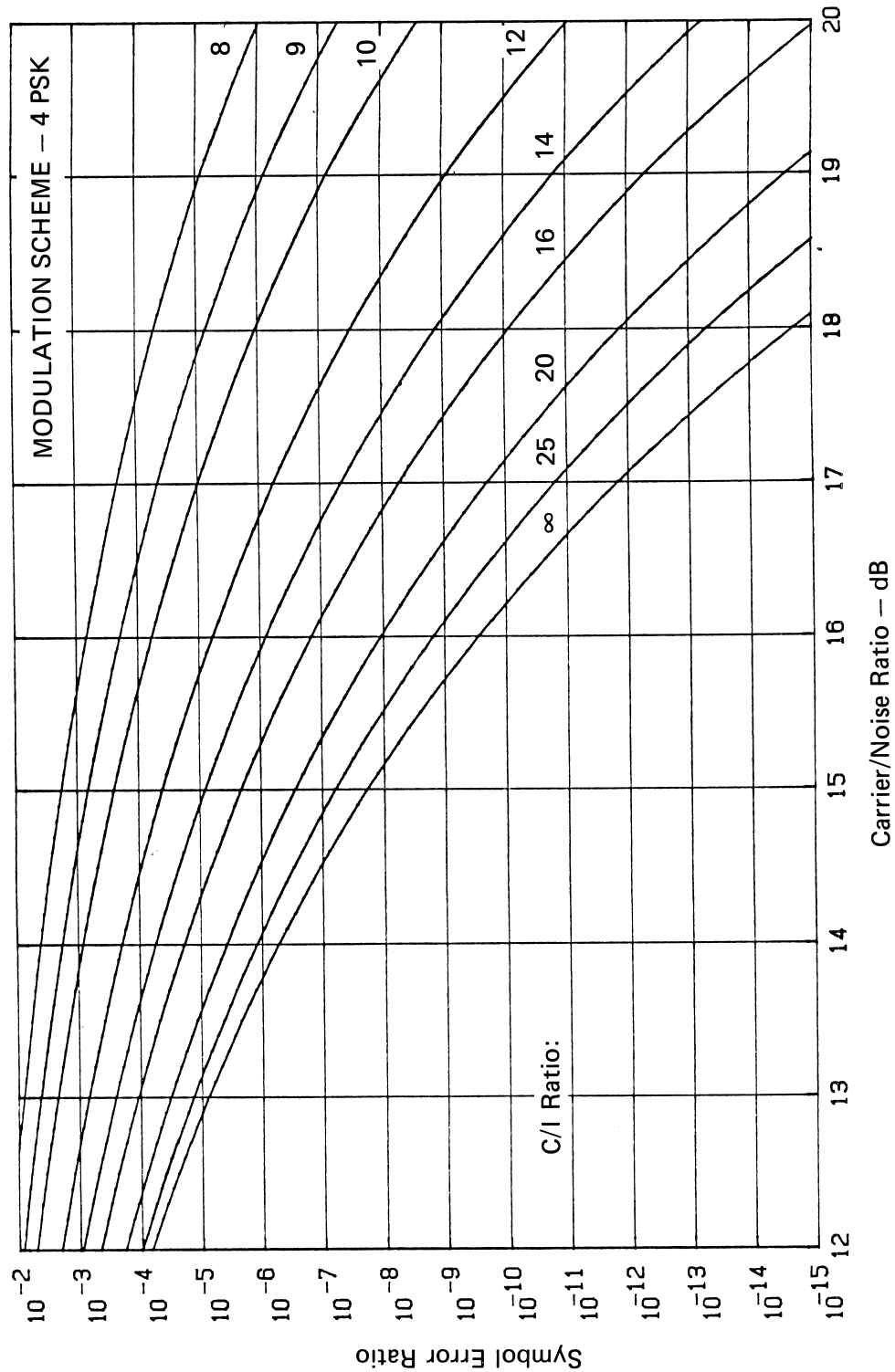
The diagram below illustrates the method:

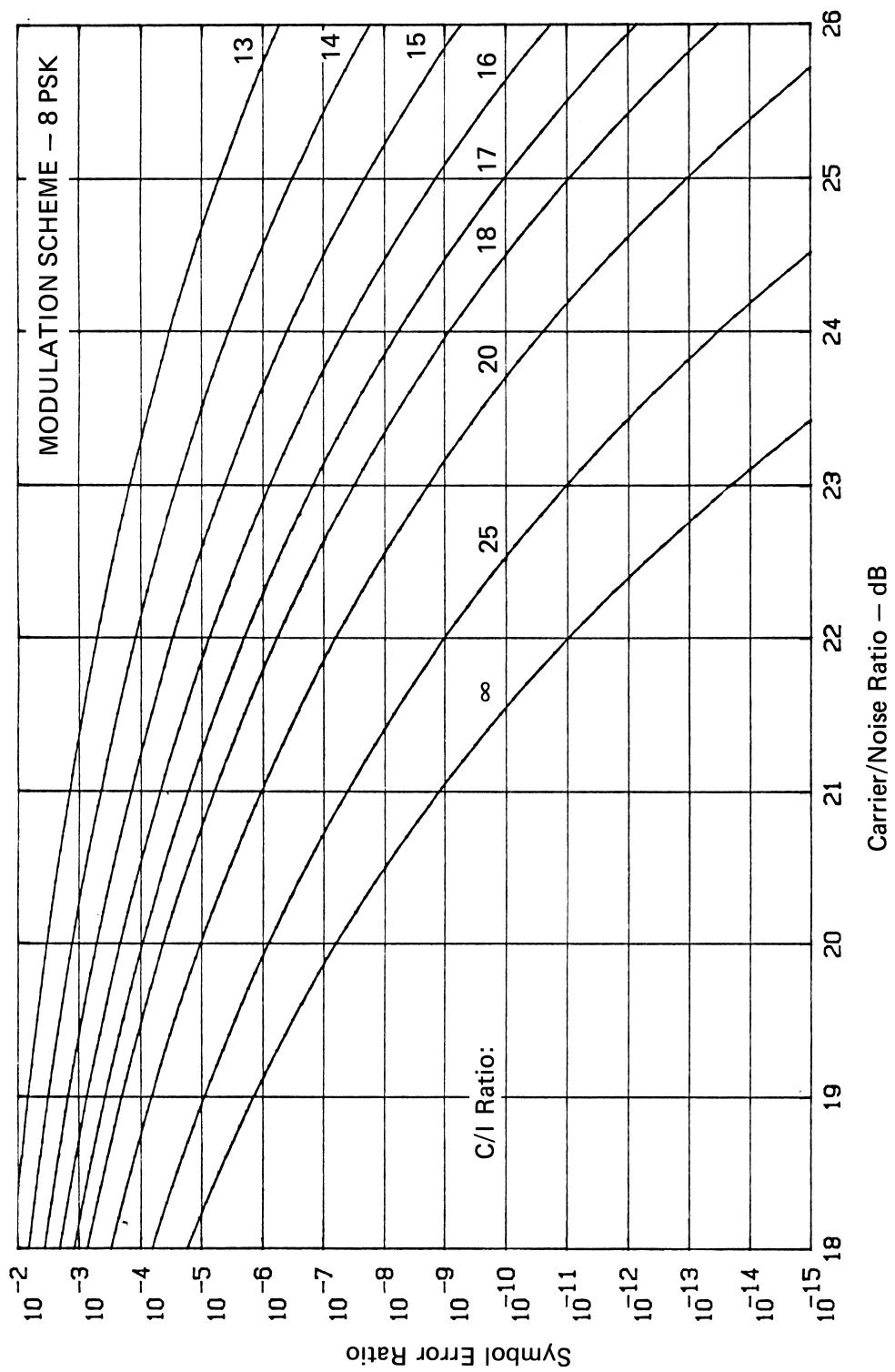


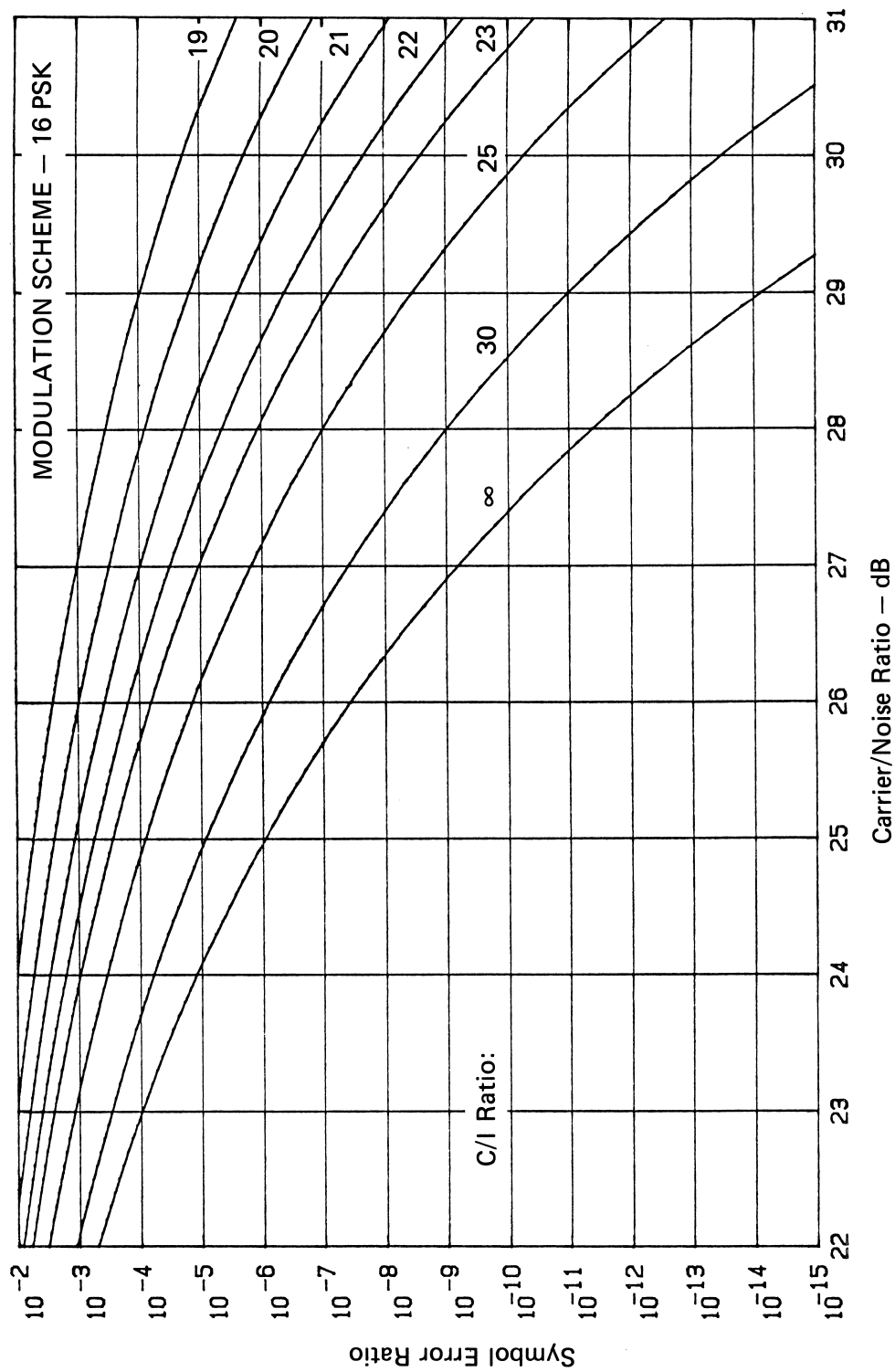
So with the addition of a simple attenuator, using the HP 3708A Option H03, the C/I Test can be performed without the need for an external signal generator, thus saving on inventory costs.

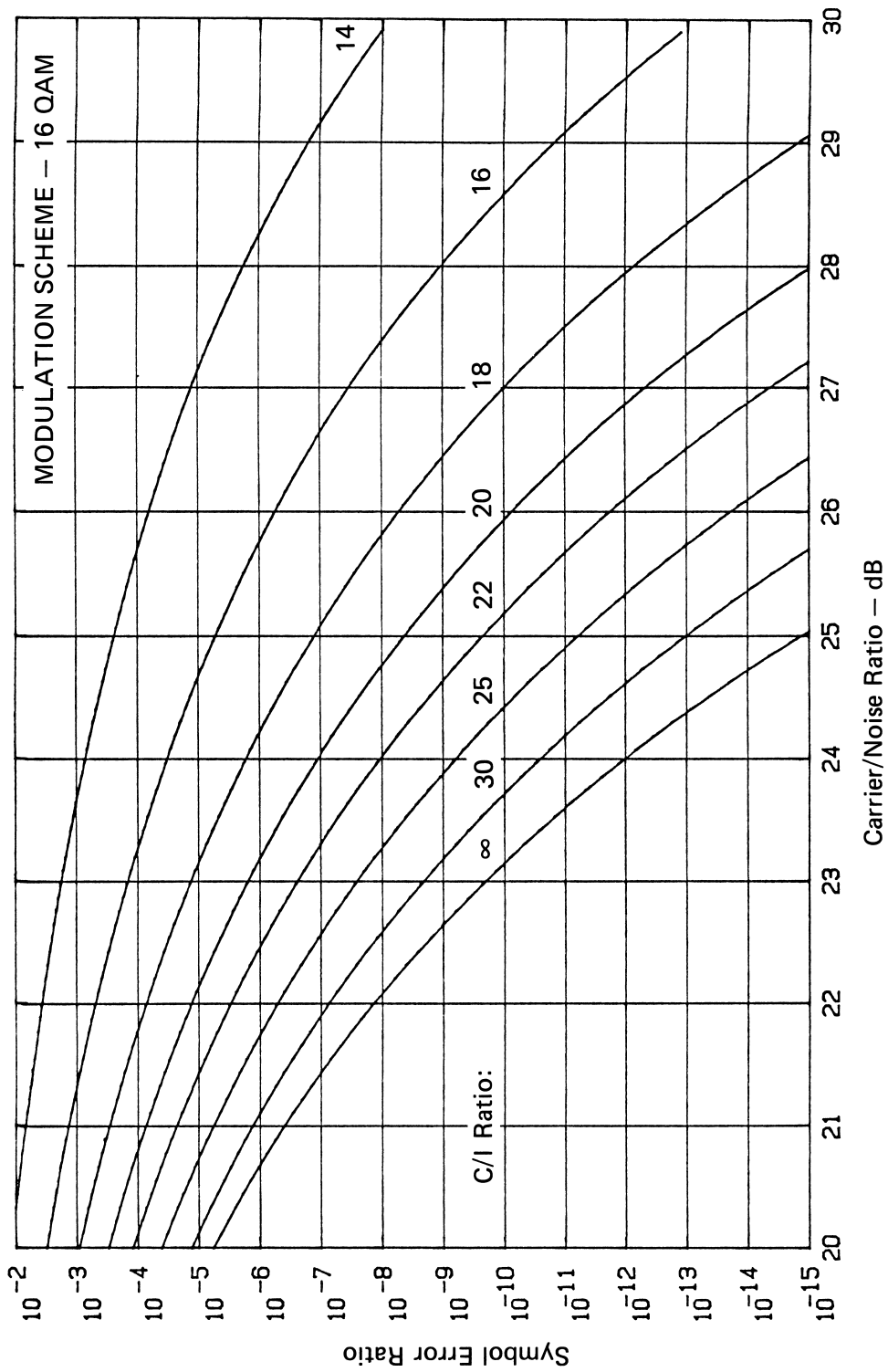
This will have particular advantages when the C/I Test is being performed in a field location.

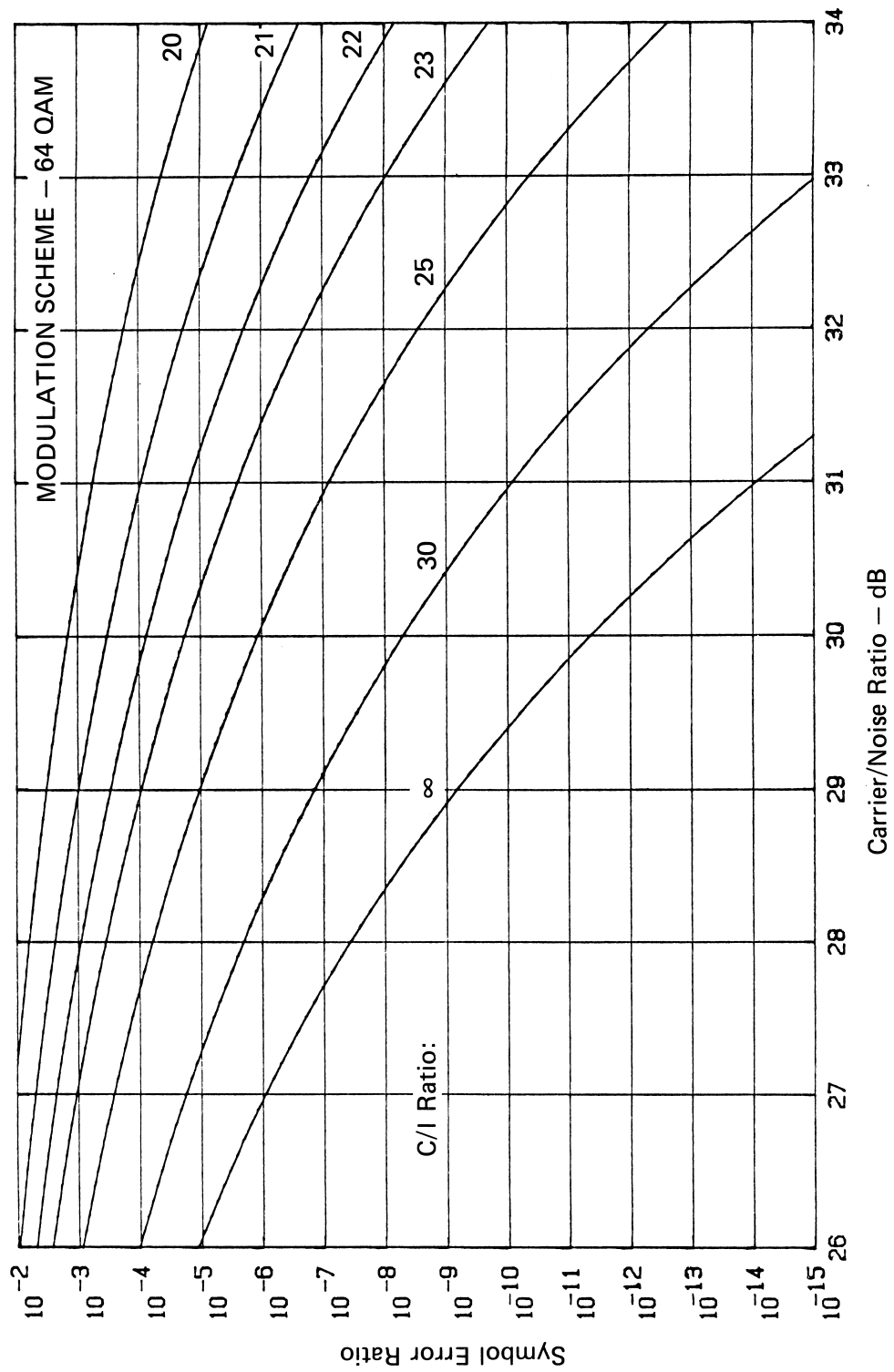
APPENDIX D: CURVES OF SYMBOL ERROR RATIO VS C/N RATIO (FOR FIXED C/I RATIOS) FOR DIFFERENT MODULATION SCHEMES











NOTES

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